Some topics/problems to study for Mathletes

1) Geometry

1) Area (triangles, squares, rectangles, rhombi, parallelograms, trapezoids, circles)
2) Perimeter/circumference (triangles, squares, rectangles, rhombi, parallelograms, trapezoids, circles)
3) Surface area—the area of all of the surfaces or sides of the 3-D object
4) Coordinate graphing
5) Angles (on a clock)

area of a circle	$A = \pi r^2$ (r = radius)
perimeter of rectangle	$P = 2l + 2w$ or $P = 2b+2h$ (l= length, w=width, b=base , h=height)
area of a rectangle and/or parallelogram	$A = bh$ (b = base, h = height)
perimeter of a square	$P = 4s$ (s=side length)
area of a square	$A = s^2$ (s = side)
area of a triangle	$A = \frac{1}{2}bh$ (b = base, h = height)
area of a trapezoid	**A= average of bases · height** or $A = \frac{1}{2}h(\text{top} + \text{bottom bases})$ (h = height)

Problem 1: A clock has a diameter of 12 inches. An ant walks from the clock's center to the numeral 2 at the edge then walks the circumference to 7, along the the shortest circumferential route, then returns to the center. How far to the nearest tenth of an inch, did the any walk? Use 3.14 for π.

Problem 2: Assemble 27 1-inch cubes into a single large cube. What portion of the original surface area is now exposed? Give answer as a fraction in lowest terms.

Problem 3: Your large pizza has a diameter of 16 inches. It is cut into 12 pieces. Your friends eat 7 pieces. What is the area of the remaining pieces? Round your answer to the nearest hundredth of a square inch. Use 3.14 for π.

Problem 4: What is the area of the figure formed by connecting the following points in order and connecting the last point back to the first? (0, 4); (1, 6); (4, 6); (5, 5); (5, 1); (0, –5); (–5, 1); (–5, 5); (–4, 6); (–1, 6)

Problem 5: What is the measurement of the acute angle formed by the hour and minute hands of a clock when it is 6:30? (Remember the hour hand will move proportionally as time passes).

2) Ratios and proportions

Notes:
proportion–An equation of two equivalent ratios.

Example: a 10 pound bag of M&Ms costs $8. How much does each pound of M&Ms cost?

$$\frac{\$8}{10 \text{ pounds}} = \frac{\$x}{1 \text{ pound}}$$

$x = \$0.80$
The M&Ms cost $0.80 per pound.

equivalent proportions–proportions that are essentially the same although they look a little different.

How can you tell if proportions are equivalent? The values that are diagonal are the same.

Example: $\frac{\$37}{100\%} = \frac{x}{70\%}$ is equivalent to $\frac{\$37}{x} = \frac{100\%}{70\%}$ and $\frac{x}{\$37} = \frac{70\%}{100\%}$

but they are **NOT** equivalent to $\frac{\$37}{x} = \frac{70\%}{100\%}$

Note: the equivalent proportions all have $37 diagonal to 70% and x diagonal to 100%. The proportion that is not equivalent does not have this quality.

Problem 6: If 3 hens lay a dozen eggs in 3 days, how long will it take one hen to lay one egg? Give your answer to the nearest hundredth of a day.

Problem 7: Suppose there are 180 people in a group. the ratio of males to females is 4: 5. Ten males leave and fifteen females arrive. What is the new ratio of males to females?

3) Palindromes

Palindrome–a sequence (word, phrase, number) that reads the same forward or backward

Problem 8: If an odometer shows 31976 miles, what is the smallest number of miles that must be traveled to make the number on the odometer a palindrome?

Problem 9:Imagine a digital clock that shows hours and minutes, and not seconds. How many palindromic times will show on the clock in a single 24-hour day?

4) Fractions

1) Ordering Fractions
2) All Operations—addition, subtraction, multiplication, and division
3) Simplifying fractions and finding equivalent fractions
4) Conversion

Problem 10: If you put three fractions between $\frac{1}{4}$ and $\frac{1}{5}$ so that all five fractions are in arithmetic sequent (i.e. the same distance apart) what will be the sum of the five fractions?

Problem 11: A field is owned by three people: Peter, Paul, and Mary. Peter owned $\frac{3}{5}$ of the field. Paul owned twice as much as Mary. What fraction of the field belongs to Mary? your answer should be in a simplified fraction.

Problem 12: Ninety people, $\frac{2}{5}$ of whom were male, were asked if they liked salmon. Forty-two of the people, including $\frac{5}{9}$ of the females said they **did** like the salmon. What fraction of the males did NOT like the salmon?

Problem 13: Bernice found the mountain bike she wanted on sale for $\frac{1}{3}$ off the original price. Because she placed her order online she received an additional 20% off the sale price. What fraction of the original price did Bernice pay for the bike? Answer as a fraction in simplest form.

5) Sequences and Patterns

Sequence– a list of numbers or objects in a special order; it is often an infinite list

Arithmetic sequence–is made by adding the same value each time

Ex. 3, 7, 11, 15…

Ex. 25, 23, 21, 19…

Geometric sequence–is made by multiplying by the same value each time.

ex. 4, 20, 100, 500, 2500…

Other sequences–several exist like the triangular number sequence (1, 3, 6, 10, 15…) or the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21…) or…

Term—an element or member of the sequence

Rule– a general formula that helps in finding the terms in the sequence

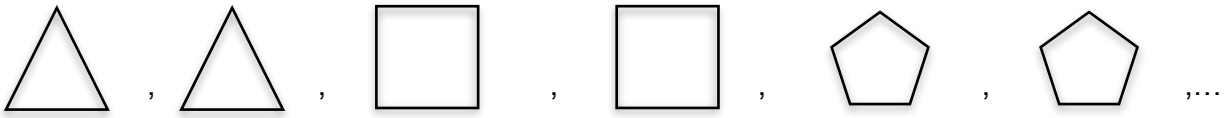
Problem 14: What is the 100th term in this sequence? 2, 6, 12, 20, 30,…

Problem 15: What is the value of the sum: 1 + 3 + 5 + 7+…+ 1997 + 1999?

Problem 16: What is the sum of the first 2016 even counting numbers? Make a simpler problem and look for a pattern.)

Problem 17: Starting with the single digit on page one, my new book has 966 digits as page numbers. All pages are numbered. How many pages long is my book?

Problem 18: The first six figures of a sequence of polygons are shown below. The seventh and eighth figures of the sequence are hexagons, and the pattern continues indefinitely. How many sides will figure number 2016 have?



Problem 42: if five hens lay 5 eggs in 5 days, then how many eggs will 100 hens lay in 100 days?

6) Writing and Solving Equations and Using Formulas

Here are some common algebraic vocabulary words:

variable- a letter that represents a number

constant- a number in an expression that doesn't change (added or subtracted)

coefficient- number multiplied by variable in an expression

expression- a mathematical phrase that contains operations, numbers, and/or variables

numerical expression- an expression that does not contain variables

algebraic expression- an expression that contains one or more variables

term- parts of expressions or equations separated by addition (+) signs.

substitution-"plugging- in" with an equivalent value

substitute-plug in with an equivalent value; directions sometimes stated as "evaluate the expression"

EXAMPLES
3x + 4 (3 is the coefficient, x is the variable, 4 is the constant, 3x + 4 is an algebraic expression, and 3x and 4 are called a terms)

$2x-5$ (2x and – 5 are the terms because it could be rewritten as 2x + – 5)

EXPRESSIONS VS. EQUATIONS

	expression	equation
	• #s	• #s
	variables	variables
	symbols (+, -, etc.)	symbols (+, -, etc.)
	• NO = sign	• = sign (yes!)
Examples:	4 + 3 6x - 1	5+ 2 = 7 3x = 9
Directions	• simplify or evaluate	solve

Example: Evaluate 5x + 4 where x = 3
substitute: 5(3)+4
simplify: = 15+4
= 19

Problem 19: Tickets to see Justin Bieber perform at the Mill Valley Middle School Fundraiser Dance cost $65 for MVMS students and $80 for non-MVMS students. The 500 tickets sell out immediately raising $34,555. How many MVMS students got tickets?

Problem 20: The florist sells single roses for $2 and bouquets of 4 roses for $6. In a day, 46 roses were sold for a total of $76. How many single roses were sold?

Problem 21: The perimeter of a triangle is 92 feet. One side of the triangle is seven feet more than twice as long as the shortest side. The third side is one foot longer than three times as long as the shortest side. What is the length of the longest side of the triangle?

Problem 22: If I gave each person 7 coins I would have 24 left. I am 32 coins short of being able to give each person 9 coins. What is the sum of people and coins?

Problem 23: Given $\boxed{ab} = ab - a - b$, find the value of b in the equation $\boxed{3b} = 5$

7) Work-Rate-Time Problems (more writing and solving equations)

There is a "trick" to doing work problems: you have to think of the problem in terms of how much each person / machine / whatever does in a given unit of time

If the problem states: "One painter can pain a house in 10 hours the other in 8 hours, how long to they take to paint the house together?"

	Person 1	Person 2
time for job completion	10 hours	8 hours
rate per unit time (hour in this case)	$\frac{1}{10} of\ the\ house\ per\ hour$	$\frac{1}{8} of\ the\ house\ per\ hour$
time	t hours	t hours

Equation: $\frac{1}{10}t+\frac{1}{8}t=1$ (rate • time for painter 1 +rate • time for painter 2 = 1 total house)

Getting a common denominator and solving gives us:

$$t=\frac{40}{9}=4\frac{4}{9}hours=4\ hours\ 26\min\ 40\ \sec$$

Problem 24: With your new lawn mower, you can mow a lawn in 4 hours. With an older mower your friend can mow the same lawn in 5 hours. Working together how long will it take both of you to mow the lawn?

Problem 25: It takes 6 hours for pump A, used alone, to fill a tank of water. Pump B used alone takes 8 hours to fill the same tank. We want to use three pumps: A, B and another pump C to fill the tank in 2 hours. What should be the rate of pump C? How long would it take pump C, used alone, to fill the tank?

Problem 26: Aaron can paint his wall in 5 hours. Lisa can paint her wall, that is twice as big (in area) as Aaron's wall, in 8 hours. If they work together to paint both of their walls, how long will the job take? Give your answer in hours and minutes.

8) Distance-Rate-Time Problems (more writing and solving equations)

Recall the* $d=r\cdot t$ *(distance = rate* $\times$ *time) formula.

Problem 27: A 700-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 120 mph. Then the tailwind picked up, and the remainder of the trip was flown at an average speed of 145 mph. For how long did the plane fly at each speed?

Problem 28: A car and a bus set out at 2 p.m. from the same point, headed in the same direction. The average speed of the car is 30 mph slower than twice the speed of the bus. In three hours, the car is 36 miles ahead of the bus. Find the rate of the car.

Problem 29: You take a 240-mile road trip. For the first 120 miles, you drive 60 miles per hour (mph). For the second 120 miles, you drive 40 mph. What is your average speed for the 240 mile road trip?

Problem 30: Two asteroids speed directly toward each other, one at 9,000 mph and one at 21,000 mph. The asteroids start 1327 miles apart. How far apart are they one minute before they collide?

9) Prime and composite numbers

prime–a number whose factors are only itself and 1 (ex. 2, 3, 5, 7, etc)
composite– a number that has more factors than itself and one (ex. 4, 6, 8, 9, 10, etc.)
Note: the number 1 is neither prime nor composite

Problem 31: Starting with the number 4 and adding 3 each time, what percentage of the first 40 numbers in the sequence are prime? Give your answer to the nearest tenth of a percent.

Problem 32: What fraction of the first ten perfect square counting numbers are one less than a prime number? Give the answer as a lowest terms fraction.

10) Factors

factor—a number or quantity that when multiplied with another produces a given number or expression
factor $\times$ factor = product

Problem 33: What is the smallest number that is a multiple of the counting numbers 2 through 14, inclusive?

Problem 34: You and a friend start running laps together. You run a lap in 12 minutes. Your friend runs a lap in 9 minutes. What will be you and your friend's combined number of laps run the first time you are back to the starting line together?

Problem 35: How many factors of 21600 are perfect squares?

11) Mean, Median, Mode, Range

To find notes on MMMR, see the Mathletes Notes webpage (on the school website) —or see Ms. Peets for a hard copy.

Problem 36: Nicole played in ten basketball games. Her average for the first two games was 8 points. Her average for the last eight games was 12 points What was her average for all 10 games?

Problem 37: The mean of four numbers is 12. When the largest of the numbers is removed, the mean of the remaining three numbers is 9. What was the number removed?

Problem 38: The sum of a whole number and the next four consecutive whole numbers is 105. Find the result when the mean of the numbers is subtracted from the median of the numbers.

12) Number Theory

Number theory or arithmetic – a branch of pure mathematics devoted primarily to the study of the integers. Number theorists study prime numbers as well as the properties of objects made out of integers (e.g., rational numbers) or defined as generalizations of the integers (e.g., algebraic integers).

Problem 39: A 5-pound box of candy can contain either 100 caramel chews or 220 molasses chips. But, I can only afford a 1/2 pound box and my Valentine likes caramel chews and molasses chips equally. If I buy a 1/2 pound box with equal weights of caramel chews and molasses chips, all together, how many pieces of candy will be in the box?

Problem 40: A forty-six year old mother has a thirteen year old daughter. In how many years will the mother be exactly twice as old as her daughter?

Problem 41: There are three consecutive odd numbers. The sum of the squares of these numbers is a 4-digit number. All 4 digits of the sum are the same. What is the smallest of the three consecutive odd integers?

Problem 42: What is the sum of the positive integers, n, for which $\frac{n+18}{n}$ is a whole number?

Problem 43: In the multiplication problem below, different letters represent different digits. What is the value of the digit ? No calculators allowed.

$$\begin{array}{r} 1ABCDE \\ \times\ 3 \\ \hline ABCDE1 \end{array}$$

13) Probability

probability-- A number that represents how likely an event is to happen.

- Probabilities range from 0% to 100%, or 0 to 1
- A probability of 0% indicates that the outcome is impossible.
- A probability of 100% indicates that the outcome must occur.
- Probabilities are usually expressed as a ratio or percent, but can be expressed as a decimal

$$\text{P(event)} = \frac{\text{\# of desired (or successful) outcomes}}{\text{\# of possible outcomes}}$$

Example

The probability of rolling doubles using two dice is: P(doubles using two dice) = $\frac{6}{36} = \frac{1}{6}$ or 50%

theoretical probability--A calculated probability based on the possible outcomes when each outcome has the same chance of occurring. It is calculated using the formula a below.

experimental probability--A probability based on data collected (in experiments). It is calculated using the formula a below.

sample space--All possible outcomes of a situation (such as heads and tails when flipping a coin)

outcome--any possible or actual result of the action considered (such as landing on heads or landing on tails when flipping a coin)

event--A desired or successful outcome or group of outcomes from an experiment, (such as landing on heads as desired when flipping a coin).

Problem 44: There are 9 red, 6 green, and 3 blue marbles in a box. How many green marbles must be added to make the probability of drawing a green marble two-thirds?

Problem 45: The probability of rain on Saturday is 60% and the probability of rain on Sunday is 50%. What is the probability of NO rain all weekend?

Problem 46: Six girls and four boys volunteered to be on a committee. If there are 2 people chosen for the committee, and they are chosen randomly, what is the probability they will both be boys?

14) Bases

To find notes on Bases, see the Mathletes Notes webpage (on the school website)—or see Ms. Peets for a hard copy.

Problem 47: What is the base 10 equivalent of the base two numeral 110101101011?

Problem 48: How is the decimal number 2017 represented in binary notation?

Problem 49: Inhabitants of the planet Hexado have three fingers on each hand. Their number system is therefore based on the powers of 6 rather than the powers of 10. Thus, they write their first 15 counting numbers in the following fashion: 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23. How is the base-ten number 2016 written on planet Hexado?

15) Percents

Problem 50: Natalie was practicing her 3-point shooting in the gym. Of her first 80 shots, she made two shots for every three shots she missed; but, now she is warmed up. How many shots must she make consecutively to bring her made shot percentage up to 50%?

Problem 51: Mr Lee had a bag of red beans and a bag of black beans. Black beans made up 20% of the total number of beans, and there were 180 oz. more red beans than black beans. Mr Lee transferred some red beans to the bag containing black beans so that the bag now contained 30% of the total number of beans. How many ounces of beans were there in the bag of mixed beans?

Problem 52: Ms. Soprano earns a commission of $1\frac{1}{2}\%$ on the selling price for each car that she sells. Last year, she sold 106 cars and earned $39, 432 in commission. What was the average selling price of the cars she sold?

16) Order of Operations

Steps:

1. parentheses (grouping)
2. exponents
3. multiplication/division (left to right)
4. addition/subtraction (left to right)

Important notes to keep in mind.

1. The "Parentheses" step includes all grouping symbols, such as (parentheses), [brackets], fraction bars separating multi-step numerators and denominators, and $|absolute\ value|$.
2. Order of operations is also followed within grouping symbols.
   Example: $\frac{3+4\cdot 5}{10-3^2}=\frac{3+20}{10-9}=\frac{23}{1}=23$ (Take note of the big fraction bar grouping symbol.)
3. When simplifying "Exponents", make sure you note whether or not the sign is being squared, cubed, or raised to a higher power.
   Examples: $(-3)^2=9$
   $-3^2=-9$
   $-(3)^2=-9$
   $0-3^2=0-9=-9$
4. Multiplication and division happen at the same time, from left to right in the problem.
5. Distributing (by multiplying) a number across parentheses is a form of multiplication.
   Examples: $9\cdot 3\div 10=27\div 10=2.7$ (multiplication first)
   $9\div 3\cdot 10=3\cdot 10=30$ (division first)
   $8(3+x)=8\cdot 3+8\cdot x=24+8x$ (distributing)
6. Taking a negative of a number is like multiplying by negative one.
   Example: $-(-5)=(-1)(-5)=5$
7. Adding and subtracting happen at the same time, starting at the left and moving to right in the problem.
   Examples: $3+7-4=10-4=6$ (adding first)
   $3-7+4=-4+4=0$ (subtracting first)

Problem 53: Evaluate: $-2^2+2(7-10)^2$ (No calculators,)

Problem 54: Evaluate: $[36\div 9+3-36+(9\div 3)]^3\div\left(1-\frac{1}{3}\right)^2=?$ (No calculators,)

Problem 55: Put a number in the blank to make this statement true: $\left(\_\_\_\_\_\div 3^2\right)^2-4+3\times 2^3=36$

17) Exponents

Example: 5^2

base: 5 exponent: 2 power: 2nd

read: five squared or five to the second power

To find more thorough notes on Exponents, see the Mathletes Notes webpage (on the school website) —or see Ms. Peets for a hard copy.

Problem 56: $2^{35}\cdot 5^{38}=1{,}250{,}000\cdot 10^x$. What is x?

Problem 57: What is the remainder when 2^{100} is divided by 5?

Problem 58: Solve for x in this equation: $\left(\frac{1}{2}\right)^{2x+8}=(4)^{2x+5}$

Please see the Mathletes web page on the school website (www. ms.mvschools.org) for more notes, the answers to the above problems (eventually), and other practice problems.